Subtraction

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This is the third in a series of five articles discussing word problems found in elementary arithmetic. In the first (published in the November/December 2010 edition of *Volta Voices*), we proposed that learning to solve such problems involves language learning as much as it involves arithmetic. In the second (published in the January/February 2011 edition of *Volta Voices*), we focused just on addition word problems. We found lots of variety in the language that they use. We encouraged parents, teachers and therapists to help children in their study of addition by exposing them to a variety of problems and by helping them to model problems, using those models to solve them.

In this article we focus on the language of subtraction word problems. As we did for addition, we can categorize them depending on how they are modeled. On that basis we can distinguish four types. The following are examples of each type:

- Easy Subtraction: Derek had 9 toy cars. He gave 5 of them to Sara. How many toy cars does Derek have left?
- Hard Subtraction 1: Derek has 9 toy cars. Sara has 5 toy cars. Derek has how many more toy cars than Sara?
- Hard Subtraction 2: Derek has 9 toy cars. Derek has 5 more toy cars than Sara has. How many toy cars does Sara have?
- Hard Subtraction 3: Derek has 9 toy cars. Sara has 5 toy cars. How many more toy cars does Sara need to have the same number as Derek?

All four of these word problems may be represented by just one equation, 9 - 5 = □. But representing their meaning requires four different models. As we noted in the case of addition, learning how to model subtraction word problems is a prerequisite to study the operation of subtraction.

The Easy Subtraction Model

To help a child understand the meaning of the Easy Subtraction example, we can provide actual toy cars (or other objects like pennies or toy blocks). We can help him or her:

- Step 1: Count out 9 objects (Figure 1a) to represent the 9 toy cars that Derek has.
- Step 2: Separate out 5 of those 9 objects (Figure 1b) to represent the toy cars that Derek gave to Sara.

The child can then use this model to solve the problem by:

- Step 3: Counting the objects that represent the toy cars that Derek has left (Figure 1c).

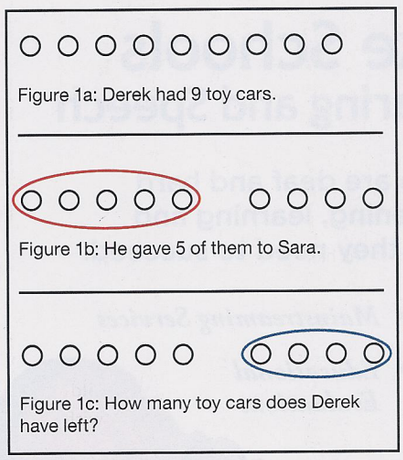

Figure 1a: Derek had 9 toy cars.

Figure 1b: He gave 5 of them to Sara.

Figure 1c: How many toy cars does Derek have left?

Here is a typical interaction as parent (Rob) presents child (Trixie) with an Easy Subtraction problem. Trixie is showing Rob her collection of pennies when Rob interrupts her.

Rob: I have a problem for you. Suppose that your friend Derek had 9 toy cars. But then Derek gave 5 of his toy cars to Sara. I want to know how many toy cars Derek would have left.

Trixie: (She tries to count out 9 of her fingers but has a hard time. She needs her right hand to point to fingers as she counts but then only has the 5 fingers of her left hand available to represent toy cars.)

Rob: Why don't you use these pennies?

Trixie: (She counts out 9 pennies and sets them aside.) 1, 2, 3, 4, 5, 6, 7, 8, 9. What was the question again?

Rob: (Pointing to her 9 pennies.) We were pretending that those 9 pennies are Derek's toy cars. But then he gives 5 of them to Sara. I wanted to know how many toy cars he will have left.

Trixie: (She counts out 5 more pennies.) 1, 2, 3, 4, 5. (But now she is confused about what Rob asked her and about what she is supposed to do.)

Rob: Wait a second. (He points to the 9 pennies that Trixie has set aside.) These are the 9 toy cars that Derek has, right? (Trixie nods.) And do you remember what happens next? (It is not clear that she does.) (Rob points to her 9 pennies.) He gave 5 of those toy cars to Sara.

Trixie: (She counts out 5 pennies, removing each one in turn from the set of 9.) 1, 2, 3, 4, 5.

Rob: Do you remember what the question was?

Trixie: No.

Rob: Derek had 9 toy cars and he gave 5 of them to Sara. How many toy cars does he have left?

Trixie: (She counts the remaining pennies.) 1, 2, 3, 4.

Rob: So there are 4 toy cars left, right?

Trixie: (She nods in agreement.)

It is clear that Trixie has several obstacles to overcome. She has trouble remembering all of the information contained in the problem. She tries to use her fingers to represent 9 toy cars, but she needs to have one hand free to point to the objects being counted. Most importantly in terms of language, she fails to understand that the 5 toy cars for Sara come from the 9 toy cars that Derek has.

Other Easy Subtraction Problems

Easy Subtraction problems can themselves be divided into several different sub-types. The language of each sub-type is significantly different from the language of all the other sub-types. While space does not permit a discussion of all of the possibilities, the examples below will help you to see the scope.[1]

- Derek had 9 toy cars. He gave some of them to Sara. Now he has 5 left. How many did he give to Sara?
- Derek had 5 toy cars. Then Sara gave him some more. Now Derek has 9 toy cars. How many toy cars did Sara give him?
- Derek had some toy cars. Then Sara gave him 5 more. Now Derek has 9 toy cars. How many toy cars did Derek have to start with?
- Derek has 9 toy cars. Five (5) of those toy cars are red and the rest are green. How many green toy cars does Derek have?

Helping children with Easy Subtraction involves helping them with the language of all these different sub-types. You should model each of these problems for yourself to see that they can all be solved using the same model as illustrated in Figure 1 on page 40. (If you are having trouble, here is a hint: start your models by representing Derek's 9 toy cars.)

Hard Subtraction 1 Model

To model the Hard Subtraction 1 problem in the Introduction:

- Step 1: Count out 9 objects (RED in Figure 2a) to represent Derek's 9 toy cars.
- Step 2: Count out 5 objects (BLUE Figure 2b) to represent Sara's toy cars.

Then solve the problem by:

- Step 3: Separating out 5 of Derek's toy cars and matching them with those that Sara has (Figure 2c).
- Step 4: Counting the extra toy cars that Derek has (Figure 2d).

Figure 2a: Derek has 9 toy cars (RED).

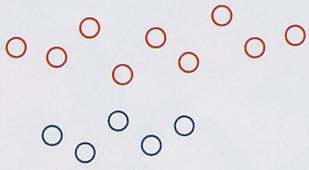

Figure 2b: Sara has 5 toy cars (BLUE).

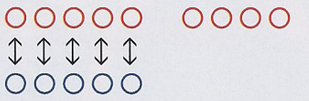

Figure 2c: Match 5 of Derek's toy cars to Sara's.

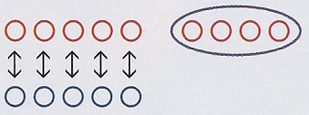

Figure 2d: Count the extra toy cars that Derek has.

This process may seem difficult to learn. And indeed, while many children learn to model Easy Subtraction without explicit instruction, Hard Subtraction problems really do cause more difficulty. But in this context it is important to remember that *if children cannot model a particular type of problem, then they don't know what problems of that type mean.* They may remember subtraction facts (in this case 9 – 5 = 4) and they may even learn to compute (e.g. 365 – 189), but they will forever have difficulty with word problems and with understanding why the computational procedures work.

Here is another example of Hard Subtraction 1:

- Derek has 9 toy cars. Sara has 5 toy cars. Sara has how many fewer toy cars than Derek?

Although the language is different, this problem has exactly the same meaning as the one modeled in Figure 2. Once again you should see that while there may only be four models for subtraction, the language of subtraction word problems is much more varied than that would suggest.

Hard Subtraction 2 and Hard Subtraction 3

Just as with Hard Subtraction 1, the examples of Hard Subtraction 2 and 3 involve the comparison of two sets of toy cars – Derek's and Sara's. We encourage you to model each of these problems for yourself. Then use those models to solve them. Doing so will force you to think very carefully about what each of the problems mean. You will see how the models differ from one another and come to a greater appreciation of what children must learn to do.

Summary

The language of subtraction word problems is even more diverse than that of addition word problems. But every subtraction word problem that children are likely to see in school can be modeled in one or the other of only four ways. Parents, teachers and listening and spoken language specialists should help children learn to model word problems, just as they help with other language learning.

Editor's Note: Past articles are available at www.JaneMadell.com and on the AG Bell website at www.agbell.org/VoltaVoices. They are also available in Spanish at www.t-oigo.com / También disponible en español en la página web, www.t-oigo.com.

1 For a more complete analysis of the language of subtraction word problems, visit www.JaneMadell.com and click on the Publications tab.