Multiplication

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This is the fourth in a series of five articles about the word problems of elementary arithmetic. In the first (published in the November/December 2010 edition of *Volta Voices*), we proposed that learning to solve such problems involves language learning as much as it involves arithmetic. In the second (published in the January/February 2011 issue of *Volta Voices*) and third (published in the March/April 2011 issue of *Volta Voices*), we focused on addition and subtraction word problems, respectively. We found lots of variety in the language of these problems. We encourage parents, teachers and therapists to help children in their study of arithmetic by exposing them to a variety of problems, helping them to make physical models of the problems and using those models to solve the problems.

In this article, we focus on the language of multiplication word problems. As we did for addition and subtraction, we can categorize multiplication problems by how they are most simply modeled. On that basis we can distinguish three types:

- Easy Multiplication: Suppose there are 3 buses and that each of those buses has 4 chickens on it. How many chickens are there altogether?
- Hard Multiplication: Matt has 4 chickens. Oscar has 3 times as many chickens as Matt. How many chickens does Oscar have?
- Combination Multiplication: Chloe likes 3 kinds of pie and 4 kinds of ice cream. If dessert consists of 1 kind of pie with 1 kind of ice cream, how many different desserts can Chloe choose from?

All three of these word problems may be represented by the one equation, 3 x 4 = □[1]. But representing their meaning is most clearly done using three different models. As we noted for both addition and subtraction, learning how to model multiplication word problems is a prerequisite to study the operation of multiplication.

Easy Multiplication

To help a child understand the meaning of the Easy Multiplication example above, try using paper cups and marbles (or any other convenient materials). Then help him or her:

- Count out 3 paper cups to represent the buses (Figure 1a).
- Put 4 marbles in each cup to represent the chickens that are on the buses (Figure 1b).

The child can then use this model to solve the problem by:

- Joining all the marbles together and counting them (Figure 1c).

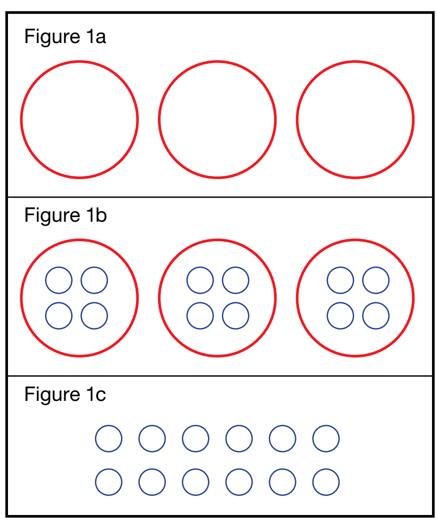


Figure 1a

Figure 1b

Figure 1c

We will return to the topic of Easy Multiplication later. But for now it is worth noting that this model of Easy Multiplication involved 15 objects. We used 3 cups to represent the buses and 12 marbles to represent the chickens.

Hard Multiplication

To help a child understand the Hard Multiplication example, we need to represent only chickens – there are no buses. And there are actually 16 chickens involved – not 12. You will need to represent all of them:

- Count out the 4 "chickens" (GREEN) that Matt has (Figure 2a).
- Help the child understand what it means for Oscar to have "3 times as many chickens" (RED). He doesn't have "just as many" (Figure 2b), or "twice as many" (Figure 2c). He has "3 times as many" (Figure 2d).
- Count how many "chickens" Oscar has: "1, 2, 3, 4...5, 6, 7, 8…9, 10, 11, 12."

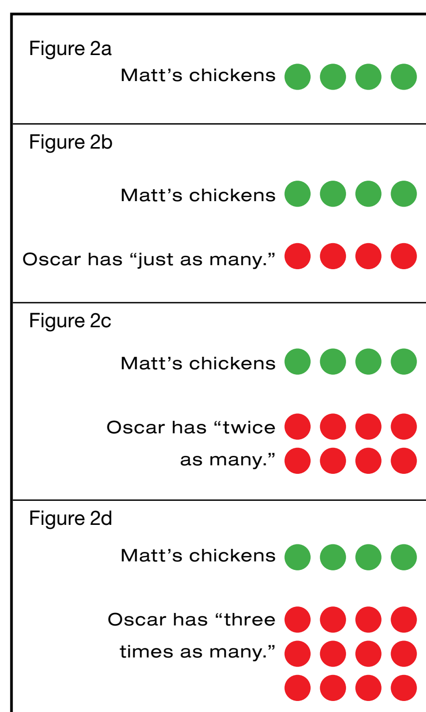


Figure 2a

Figure 2b

Figure 2c

Figure 2d

1 The symbols "3 x 4" *means* 3 collections of objects, with 4 objects in each collection. The symbols "4 x 3" *means* 4 collections of objects, with 3 objects in each collection. These meanings are different from one another. Using the models in the discussion that follows, you should be able to see that both the Easy Multiplication example and the Hard Multiplication example should be represented by 3 x 4 = □, but not by 4 x 3 = □. On the other hand, you will also be able to see that the Combination Problem can be equally well represented by *both* 3 x 4 = □ and 4 x 3 = □.

While many children learn to model Easy Multiplication problems without explicit instruction, that is often not the case for Hard Multiplication. This may be a good time to again emphasize the importance of teaching children to make models of all the different kinds of word problems that they are likely to encounter, first as students in school and then as adults in supermarkets, gas stations and the workplace.

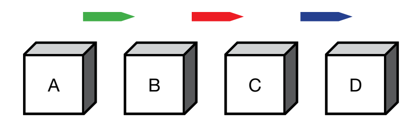
Some things to remember: 1) If children cannot model a particular type of problem and solve problems of that type by counting, then they don't know what problems of that type mean. 2) The reason to memorize so-called "facts" (like 3x4=12) is to allow simple word problems to be solved quickly, without models and without counting. But the experience of making models and counting helps children learn to use those facts to solve word problems. And it is only with that experience that they can see the value of memorization. 3) Finally, the experience of making models and counting is necessary if children are to understand why the familiar rules for adding, subtracting, multiplying and dividing actually work.

Combination Multiplication

Children only rarely learn to model Combination Multiplication problems on their own. But our experience suggests that this is not difficult to teach. However, the modeling and counting present some new challenges. You may find it helpful to make your own physical model as you read along.

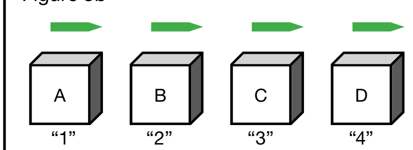
- Count out 3 markers to represent the 3 kinds of pie – perhaps 3 crayons of different colors. Then count out 4 markers to represent the 4 kinds of ice cream – perhaps 4 toy blocks with different letters on them (Figure 3a). (Challenge 1: Unlike any of the problems modeled so far, it will be helpful if the 3 kinds of "pie" are similar, e.g., all crayons, but distinguishable from one another, e.g., different colors. The 4 kinds of "ice cream" should also be similar – but distinguishable – from one another and also distinguishable from the "pie.")

Figure 3a



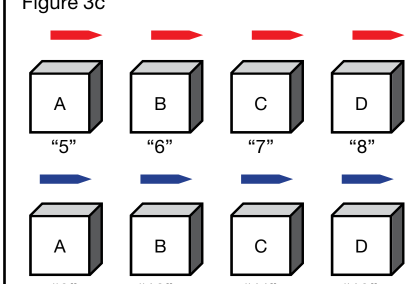
- Combine one of the "pies" (in this case the GREEN crayon) with each of the 4 blocks representing the different kinds of ice cream. As you make these combinations, keep track of how many you have made (Figure 3b). (Challenge 2: You cannot make all 4 combinations at once since you only have one green crayon. Therefore, you will need to count as you go along, and remember how many you have counted so far.)

Figure 3b



- Next, combine the RED "pie" with each of the different kinds of "ice cream" and the BLUE "pie" with each of the different kinds of "ice cream." As you keep making these combinations, keep track of how many you have made (Figure 3c). (Challenge 3: You need a system for keeping track of the combinations, counting each one once and only once.) Do you see why this problem can be represented by both $3 \times 4 = \square$ and $4 \times 3 = \square$?

Figure 3c



Other Multiplication Word Problems

While there are only three models for the simple multiplication problems that children are likely to see in school, the language of multiplication problems is more diverse. For example, among those problems that can be modeled as Easy Multiplication, mathematics educators have distinguished "Price Problems," "Rate Problems" and "Array Problems":

- Price Problems – *Isabella buys 3 candy bars. Each one costs 4 cents. How much do the candy bars cost altogether?*
- Rate Problems – *Jacob plants 4 sunflower seeds each day. How many sunflower seeds does he plant in 3 days?*
- Array Problems – *The seats in a small classroom are arranged in 3 rows, with 4 seats in each row. How many seats are there altogether?*

We leave it to the reader to model these problems. See if you can satisfy yourself that these problems are just examples of Easy Multiplication.

Summary

Once again we emphasize the main point of this series. Parents, teachers and listening and spoken language specialists should help children learn to model all the different kinds of word problems, just as they help with other types of language learning. In this article we identified the models for multiplication problems and some of the language that gives rise to them.

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