Addition

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This is the second in a series of articles about the word problems of elementary arithmetic. In the first (published in the November/December 2010 issue of *Volta Voices*), we tried to show that learning to solve such problems involves language learning as much as it involves arithmetic. Our purpose here is to carefully examine the language of addition word problems. (In subsequent articles we will examine the language of subtraction, multiplication and division.) Parents and teachers, as well as speech-language pathologists and Listening and Spoken Language Specialists, in the case of children who are deaf or hard of hearing, should be prepared to help children acquire this language.

Everyone is familiar with word problems – from school arithmetic and from daily life. Here are two simple examples:

Problem 1: Eva has 4 lollipops. Rose has 5 lollipops. How many lollipops do they have altogether?

Problem 2: Eva has 4 lollipops. Rose has 5 more lollipops than Eva has. How many lollipops does Rose have?

Most adults realize that both of these word problems may be represented by the equation 4 + 5 = □. But in spite of this similarity you will see below that the two problems have distinct models. Both of those models are important in the study of arithmetic. Importantly, the ability to model word problems involves learning language.

A Model for "Easy Addition"

Problem 1 is an example of what we call "Easy Addition." A model is shown in Figure 1.

To help a child represent Problem 1, we can provide actual lollipops

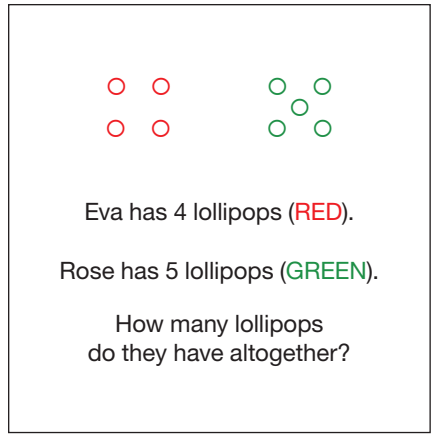

Figure 1 – Easy Addition

(or other objects like pennies or toy blocks). Then we can help him or her:

- Count out 4 objects (RED) to represent Eva's 4 lollipops.
- Count out 5 objects (GREEN) to represent Rose's 5 lollipops.

He or she can then be guided to use this model to solve the problem by:

- Moving the two collections of objects together.
- Counting the two collections together (1, 2, 3, 4, 5, 6, 7, 8, 9).
- Reporting the answer, "9."

The child who can do this on his or her own understands the language of the problem and has a beginning understanding of addition.

A Model for "Hard Addition"

Problem 2 is an example of what we call "Hard Addition." A model for it is shown in Figure 2.

To help a child model Problem 2, and use the model to solve it, we can use the following process:

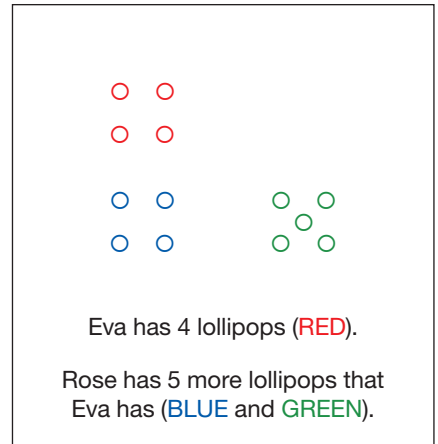

Figure 2 – Hard Addition

- Count out 4 objects (RED) to represent Eva's 4 lollipops.
- Count out another 4 objects (BLUE) to represent Rose's lollipops that correspond to Eva's 4 lollipops.
- Count out 5 more objects (GREEN) to represent Rose's "5 more" lollipops.
- Move Rose's two collections of objects together.
- Count Rose's two collections together (1, 2, 3, 4, 5, 6, 7, 8, 9).
- And report the answer, "9."

This is not the place for a thorough discussion of the role that the ability to model word problems has in the study of arithmetic. Suffice it to say that this ability is prerequisite to study of the operation of addition. For example, without such understanding it is impossible to understand why the rules for adding a column of figures make sense and actually work for solving word

problems.[1] The important point for our purposes here is that *at this stage, learning arithmetic and the ability to understand the language of and model word problems are one and the same thing.*

Other Addition Word Problems

Every addition word problem that children are likely to see in school can be modeled by one of the structures described above. That is, each such problem is an example of either Easy Addition or Hard Addition. But that is not to say that there aren't significant differences of language among Easy Addition problems and among Hard Addition problems. Consider the following:

> Problem 3: Eva had some lollipops. Eva gave 5 of her lollipops to Rose. If Eva now has 4 lollipops left, how many lollipops did she start out with?

To help a child solve this problem you could help him or her build and then use the model shown in Figure 3.

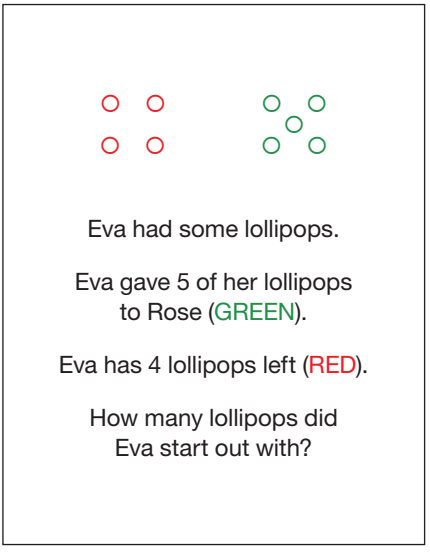

Figure 3

The model for Problem 3 is the same as that for Problem 1 (Figure 1) – it is, therefore, another example of Easy Addition. But while many children come to understand the language of Problem 1 without formal instruction, most will have difficulty with Problem 3. It is important that children understand both because one of the most important goals of instruction in arithmetic is that children learn to solve all the different kinds of word problems that surround us in our daily lives.

Some mathematics educators distinguish a third category of Easy Addition problems:

> Problem 4: Eva had 4 lollipops. Rose gave her 5 more lollipops. How many lollipops does Eva have now?

Although it is very similar to Problem 1, Problem 4 explicitly describes the joining of the two sets of lollipops. By contrast, the situation described in Problem 1 is static – there is no change in the situation over time. In spite of this distinction, it has been our experience that for instructional purposes, these two categories of Easy Addition problems are essentially equivalent. That is, there is little difference between helping a child model Problem 1 and helping him or her model Problem 4.

Finally, here is a second example of Hard Addition:

> Problem 5: Eva has 4 lollipops. Eva has 5 fewer lollipops than Rose. How many lollipops does Rose have?

Although the meaning of Problem 5 is identical to that of Problem 2 (and their models are identical), you won't be surprised that the language of Problem 5 causes more difficulties.

Conclusion

There is significant variety in the language of addition word problems. But the ones that children see in school are all either "Easy" or "Hard." Parents, teachers and therapists can help children in their study of addition by exposing them to both types, by helping them to model the problems and by helping them to use those models to solve them.

The authors would like to thank Rosemary Brener, Ph.D., for useful conversations during the development of this article.

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1 Van de Walle, J.A. (2004). *Elementary and Middle School Mathematics, Fifth Edition*. Upper Saddle River, N.J.: Pearson Education Inc.